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Is the Electromagnetic Field an Invariant Superposition of the 42 States of a Basic Field?

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Abstract

It is shown that an invariant superposition of 42 symmetry-related states of a basic field car account for the experimental value of the fine-structure constant. Such a theory yields the formula $\hbar c/e^2 = N(N-1)/4\pi = 137.0324$, with $N = 42$ the order of the invariance group.

Associated with the electromagnetic four-potential $a_\mu(x)$, we have the rescaled field $\xi_\mu(x) \equiv e^{-1}a_\mu(x)$ with inverse-length dimensions, where e is the fundamental unit of electrical charge; the rescaled field components are $\xi_0(x) = (x_1{}^2 + x_2{}^2 + x_3{}^2)^{-1/2}$, $\xi_i(x) \equiv 0$ in the vicinity of a proton at rest at the origin of the spatial coordinates. Quantization of the electromagnetic field brings in the fundamental commutator

$$[a_\mu(x), a_\nu(y)] = 4\pi i\hbar c D(x-y)g_{\mu\nu} \tag{1}$$

where $D(x)$ is the Lorentz-invariant geometrical distribution function and $g_{\mu\nu}$ is the Minkowski metric tensor. By putting $a_\mu(x) = e\xi_\mu(x)$ into (1) and using the experimental value $\hbar c/e^2 = 137.0360$, we obtain the fundamental commutator as a statement involving geometrically dimensioned quantities exclusively:

$$[\xi_\mu(x), \xi_\nu(y)] = 1722.045 iD(x-y)g_{\mu\nu} \tag{2}$$

The small deviation of the numerical coefficient in (2) from the integer value 1722, amounting to 26 parts per million, may be due to physical charge renormalization effects or to an inaccuracy of magnitude 0.0036 in the current experimental value for $\hbar c/e^2$. If the numerical coefficient is precisely 1722 in the fundamental commutator, we have

$$[\xi_\mu(x), \xi_\nu(y)] = N(N-1)iD(x-y)g_{\mu\nu} \tag{3}$$

with $N = 42$.

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TABLE 1. Properties of the transitive subgroup of order 42 contained in the symmetric group of degree 7

Cycles: conjugate classes	1^7	1, 6	1, 6	$1, 2^3$	$1, 3^2$	$1, 3^2$	7
Order: number of elements	1	7	7	7	7	7	6
Characters of faithful rep: Trace of M_r's	6	0	0	0	0	0	−1

How can the integer 42 enter a more fundamental theory for quantum electrodynamics? $N = 42$ is the order of a transitive subgroup of the symmetric group of degree 7, a subgroup associated with restricted permutations on 7 objects (Littlewood, 1940) (see Table 1). There is only one faithful irreducible representation for this group of order 42, and it is the six-dimensional representation obtained in the $\mathbf{6} \oplus \mathbf{1}$ reduction of the 42 seven-dimensional permutation matrices. Letting v denote a six-tuple in the basis space and $\{M_1 \equiv 1, M_2, \ldots, M_{42}\}$ the six-dimensional matrices in the faithful representation, we have 42 related field-theoretic states

$$\xi_\mu^{(r)}(x; v) \equiv \xi_\mu^{(1)}(x; M_r v), \qquad r = 1, \ldots, 42 \tag{4}$$

associated with a field $\xi_\mu^{(1)}(x; v)$ that is labeled by quantum numbers in the six-tuple v. The sum of all of the symmetry-related states in (4),

$$\xi_\mu(x) \equiv \sum_{r=1}^{42} \xi_\mu^{(r)}(x; v) \tag{5}$$

is invariant under the 42 transformations $v \to M_s v$ for $s = 1, \ldots, 42$. Also invariant under the 42 permutation symmetry operations are the commutators

$$[\xi_\mu^{(r)}(x), \xi_\nu^{(s)}(y)] = \begin{cases} iD(x-y)g_{\mu\nu} & \text{for} \quad r \neq s \\ 0 & \text{for} \quad r = s \end{cases} \tag{6}$$

From (5) and (6) we get

$$\begin{aligned}[\xi_\mu(x), \xi_\nu(x)] &= \sum_{r,s=1}^{N} [\xi_\mu^{(r)}(x), \xi_\nu^{(s)}(y)] \\ &= N(N-1)iD(x-y)g_{\mu\nu}\end{aligned} \tag{7}$$

and hence the rescaled electromagnetic field in (3) can be identified with the group-invariant superposition in (5). Although it would be more usual to have the $r \neq s$ and $r = s$ statements interchanged, the commutators (6) are dynamically

TABLE 2. Hypercharge and isospin content of the $(p_1, p_2) = (2, 3)$ irreducible representation of $SU(3)$. For each value of Y, the allowable values of I satisfy the bounding relation $|\frac{1}{2}Y + \frac{1}{3}| \leq I \leq \frac{5}{2} - |\frac{1}{2}Y - \frac{1}{6}|$.

Hypercharge, Y	$\frac{7}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{5}{3}$	$-\frac{8}{3}$
Isospin, Y	$\frac{3}{2}$	2, 1	$\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$	2, 1, 0	$\frac{3}{2}, \frac{1}{2}$	1
Total number of states with Y value	4	8	12	9	6	3

consistent and can in fact be derived from a canonical theory based on the Lagrangian

$$\mathscr{L} = \tfrac{1}{2}\hbar \left[-(N-1)^{-1} \xi_{\mu,\nu} \xi^{\mu,\nu} + \sum_{r=1}^{N} \xi^{(r)}_{\mu,\nu} \xi^{(r)\mu,\nu} \right] \tag{8}$$

in which the 42 $\xi^{(r)}_\mu$'s are treated as independent fields with ξ_μ dependent on them and defined by (5).

What physical significance can be assigned to the six-tuple of quantum number labels denoted by v in (4) and to the 42 related field-theoretic states that superimpose additively according to (5) to produce the group-invariant electromagnetic field? The dimension of an irreducible representation of $SU(3)$ is given by

$$N = \tfrac{1}{2}(p_1 + 1)(p_1 + p_2 + 2)(p_2 + 1) \tag{9}$$

in which p_1 and p_2 are the number of covariant and the number of contravariant indices, respectively, on the basis tensor. Hence, for either $(p_1, p_2) = (2, 3)$ or $(3, 2)$ we have an $N = 42$ irreducible representation. The hypercharge and isospin content of either $N = 42$ irreducible representation follows immediately from the weight diagram and is shown in Table 2 for $(p_1, p_2) = (2, 3)$. It is a simple matter to prescribe a correspondence between the 42 states labeled by allowable values of Y, I, I_3 and the 42 six-tuples $M_s v$ with $s = 1, \ldots, 42$ and v fixed conveniently. Clearly, for such a correspondence the superposition of symmetry-related fields (5) does not admit a sharp value for Y or I_1, I_2, I_3 but gives the expectation values $\langle Y \rangle = \langle I_i \rangle = 0$. The latter hypercharge and isospin properties are featured by the one-photon and multiphoton quantum states of the electromagnetic field, and thus the rescaled electromagnetic field (5) may be interpreted physically in terms of an $SU(3)$-breaking permutation symmetric superposition of all states in a 42-dimensional multiplet.

Reference

Littlewood, D. E. (1940). *Theory of Group Characters*, p. 276. Oxford University Press, London.